Indian Statistical Institute
B. Math. (Hons.) III Year
I Semestral Examination 2010-2011
Differential Geometry (Back Paper)

Duration: 3 Hours   Marks: 100   Instructor: B. Bagchi

1. (a) Let γ be a unit speed convex simple closed curve say with length l. Use the equation $n_s' = -k_s t$ to show that the signed curvature k_s of γ satisfies $\int\limits_0^l k_s'(t)\gamma(t)dt = 0$.

   (b) Use part (a) to prove the four vertex theorem.

   (c) Show that any ellipse (which is not a circle) has at least four vertices.

   [10 + 15 + 5 = 30]

2. Let $((a_{ij}))$ be a 3×3 skew symmetric matrix whose entries are smooth functions of a real parameter t. Let v_1, v_2, v_3 be smooth solutions of the differential equation

$$v_i'(t) = \sum_{j=1}^{3} a_{ij}\ v_j(t), 1 \leq i \leq 3.$$

   Show that if $\{v_i(t) : 1 \leq i \leq 3\}$ is an orthonormal basic for $\mathbb{R}^3$ at some point t, then the same is true at all points. [25]

3. Let $f : S_1 \longrightarrow S_2$ be a diffeomorphism between two smooth surfaces. Show that

   (a) f is an isometry iff any two co-ordinate charts of S_1, S_2 which correspond under f ($i \cdot e,\ \sigma_2 = f \circ \sigma_1$) have the same first fundamental form.

   (b) State and prove a similar criterion for f to be conformal.

   [15 + 15 = 30]

4. State the formula for the area of a spherical triangle in terms of its angles. Use this formula to prove that $V - E + F = 2$ for any triangulation of the sphere. [5 + 10 = 15]